

S-wave $\gamma\gamma \rightarrow \pi\pi$ and $f_0(980) \rightarrow \pi\pi$

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Abstract. We report on a dispersion relation for the $\gamma\gamma \rightarrow (\pi\pi)_I$ S-wave in isospin I emphasizing the low energy region. The $f_0(980)$ signal that emerges in $\gamma\gamma \rightarrow \pi\pi$ is also discussed. Our results could be used to distinguish between different $\pi\pi$ isoscalar S-wave parameterizations. We also calculate the width of the σ resonance to $\gamma\gamma$ and obtain the value $\Gamma(\sigma \rightarrow \gamma\gamma) = (1.68 \pm 0.15)$ KeV. Finally, we elaborate on the size of the $f_0(980)$ coupling to $\pi\pi$ and show that its smallness compared to the $K\bar{K}$ one is not related to the OZI rule.

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INTRODUCTION

The study of the reaction $\gamma\gamma \rightarrow \pi^0\pi^0$ offers the possibility of having two neutral pions in the final state as a two body hadronic system. Because $\gamma\gamma \rightarrow \pi^0\pi^0$ has no Born term the final state interactions dominate this reaction. In this respect, one can think of the interest of precise data on this process to test present low energy parameterizations of the $\pi\pi$ isospin (I) 0 S-wave, to further study the nature of the σ resonance as well as that of the $f_0(980)$, and having another way to constraint their pole positions [1, 2]. The σ coupling to $\gamma\gamma$ is discussed and we obtain $\Gamma(\sigma \rightarrow \gamma\gamma) = (1.68 \pm 0.15)$ KeV. Finally, we show that the suppression of the $f_0(980)$ coupling to $\pi\pi$ in comparison with $K\bar{K}$ is not due to the OZI rule, as one could be tempted to think [3].

$$\gamma\gamma \rightarrow \pi\pi$$

For this and the next section we report on the results of refs.[4, 5], where a more detailed account can be found. Let us consider the S-wave amplitude $\gamma\gamma \rightarrow (\pi\pi)_I$, $F_I(s)$, where the two pions have definite $I = 0$ or 2 . The function $F_I(s)$ on the complex s -plane is analytic except for two cuts along the real s -axis, the unitarity one for $s \geq 4m_\pi^2$ and the left hand cut for $s \leq 0$, with m_π the pion mass. Let us denote by $L_I(s)$ the complete left hand cut contribution to $F_I(s)$. Then, the function $F_I(s) - L_I(s)$, by construction, has only right hand cut. Let $\phi_I(s)$ be the phase of $F_I(s)$ modulo π , chosen in such a way that $\phi_I(s)$ is *continuous* and $\phi_I(4m_\pi^2) = 0$. For the exotic $I = 2$ S-wave one can invoke Watson's

final state theorem¹ so that $\phi_2(s) = \delta_\pi(s)_2$. For $I = 0$ the same theorem guarantees that $\phi_0(s) = \delta_\pi(s)_0$ for $s \leq 4m_K^2$, where we denote by $\delta_\pi(s)_I$ the isospin I S-wave $\pi\pi$ phase shifts. Here one neglects the inelasticity due to the 4π and 6π states below the two kaon threshold $s_K = 4m_K^2$, the phase function $\phi_0(s)$ cannot be fixed *a priori* due the onset of inelasticity. However, as remarked in refs.[7, 8], inelasticity is again small for $\sqrt{s} \gtrsim 1.1$ GeV [6], and one can then apply approximately Watson's final state theorem which implies that $\phi_0(s) \simeq \delta^{(+)}(s)$ modulo π . Here $\delta^{(+)}(s)$ is the eigenphase of the $\pi\pi$, $K\bar{K}$ $I = 0$ S-wave S-matrix such that it is continuous and $\delta^{(+)}(s_K) = \delta_\pi(s_K)_0$. In refs.[9, 8] it is shown that $\delta^{(+)}(s) \simeq \delta_\pi(s)_0$ or $\delta_\pi(s)_0 - \pi$, depending on whether $\delta_\pi(s_K)_0 \geq \pi$ or $< \pi$, respectively. In order to fix the integer factor in front of π in the relation $\phi_0(s) \simeq \delta^{(+)}(s)$ modulo π , it is necessary to follow the possible trajectories of $\phi_0(s)$ in the *narrow* region $1 \lesssim \sqrt{s} \lesssim 1.1$ GeV. The remarkable physical effects happening there are the appearance of the $f_0(980)$ resonance on top of the $K\bar{K}$ threshold and the cusp effect of the latter that induces a discontinuity at s_K in the derivative of observables. Between 1.05 to 1.1 GeV there are no further narrow structures and observables evolve smoothly. Approximately half of the region between 0.95 and 1.05 GeV is elastic and $\phi_0(s) = \delta_\pi(s)_0$ (Watson's theorem), so that it raises rapidly. Above $2m_K \simeq 1$ GeV and up to 1.05 GeV the function $\phi_0(s)$ can keep increasing with energy, like $\delta_\pi(s)_0$. The other possibility is a change of sign in the slope at s_K due to the $K\bar{K}$ cusp effect such that $\phi_0(s)$ starts a rapid decrease in energy. Above $\sqrt{s} = 1.05$ GeV, $\phi_0(s)$ matches smoothly with the behaviour for $\sqrt{s} \gtrsim 1.1$ GeV, which is constraint by Watson's final state theorem. As a result, for $\sqrt{s} \gtrsim 1$ GeV *either* $\phi_0(s) \simeq \delta_\pi(s)_0$ *or* $\phi_0(s) \simeq \delta_\pi(s) - \pi$, corresponding to an increasing or decreasing $\phi_0(s)$ above s_K , in order.

Let us define the switch z to characterize the behaviour of $\phi_0(s)$ for $s > s_K$, and close to s_K , such that $z = +1$ if $\phi_0(s)$ rises with energy and $z = -1$ if it decreases. Let s_1 be the value of s at which $\phi_0(s_1) = \pi$. Following ref.[8] we introduce the Omnès functions,

$$\begin{aligned}\Omega_0(s) &= \left(1 - \theta(z) \frac{s}{s_1}\right) \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\phi_0(s')}{s'(s' - s)} ds' \right], \\ \Omega_2(s) &= \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\phi_2(s')}{s'(s' - s)} ds' \right].\end{aligned}\tag{1}$$

with $\theta(z) = 1$ for $z = +1$ and 0 for $z = -1$. Given the definition of the phase function $\phi_I(s)$ the function $F_I(s)/\Omega_I(s)$ has no right hand cut. Next, we perform a twice subtracted dispersion relation for $(F_0(s) - L_0(s))/\Omega_0(s)$

$$F_0(s) = L_0(s) + c_0 s \Omega_0(s) + \frac{s^2}{\pi} \Omega_0(s) \int_{4m_\pi^2}^{\infty} \frac{L_0(s') \sin \bar{\phi}_0(s')}{s'^2 (s' - s) |\Omega_0(s')|} ds' + \theta(z) \frac{\omega_0(s)}{\omega_0(s_1)} \frac{s^2}{s_1^2} (F_0(s_1) - L_0(s_1)),\tag{2}$$

¹ This theorem implies that the phase of $F_I(s)$ when there is no inelasticity is the same, modulo π , as the one of the isospin I S-wave $\pi\pi$ elastic strong amplitude.

where $\omega_0(s) = \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\phi_0(s')}{s'(s'-s)} ds' \right]$ [10]. In the previous equation we introduce $\bar{\phi}_0(s)$ that is defined as the phase of $\Omega_0(s)$. Proceeding similarly for $I = 2$ one has

$$F_2(s) = L_2(s) + c_2 s \Omega_2(s) + \frac{s^2}{\pi} \Omega_2(s) \int_{4m_\pi^2}^{\infty} \frac{L_2(s') \sin \phi_2(s')}{s'^2(s'-s) |\Omega_2(s')|} ds' . \quad (3)$$

It is worth mentioning that eq.(2) for $I = 0$ and $z = +1$ is equivalent to perform a three times subtracted dispersion relation for $(F_0(s) - L_0(s))/\omega_0(s)$,

$$F_0(s) = L_0(s) + c_0 s w_0(s) + d_0 s^2 w_0(s) + \frac{s^3 w_0(s)}{\pi} \int_{4m_\pi^2}^{\infty} \frac{L_0(s') \sin \phi_0(s')}{s'^3(s'-s) |\omega_0(s')|} ds' . \quad (4)$$

Let us denote by $F_N(s)$ the S-wave $\gamma\gamma \rightarrow \pi^0 \pi^0$ amplitude and by $F_C(s)$ the $\gamma\gamma \rightarrow \pi^+ \pi^-$ one. The relation between F_0 , F_2 and $F_N(s)$, $F_C(s)$ is

$$F_N(s) = -\frac{1}{\sqrt{3}} F_0 + \sqrt{\frac{2}{3}} F_2 , \quad F_C(s) = -\frac{1}{\sqrt{3}} F_0 - \sqrt{\frac{1}{6}} F_2 . \quad (5)$$

We are still left with the unknown subtraction constants c_0 , c_2 for $I = 0$ and 2, respectively, and $F_0(s_1) - L_0(s_1)$ for $I = 0$ and $z = +1$. In order to determine them we impose the following conditions:

1. $F_C(s) - B_C(s)$ vanishes linearly in s for $s \rightarrow 0$ and we match the coefficient to the one loop χ PT result [11, 12]. Here B_C is the Bron term for $\gamma\gamma \rightarrow \pi^+ \pi^-$.
2. $F_N(s)$ vanishes linearly for $s \rightarrow 0$ and the coefficient can be obtained again from one loop χ PT [11, 12].
3. For $I = 0$ and $z = +1$ one has in addition the constant $F_0(s_1) - L_0(s_1)$. Its value can be restricted because the cross section $\sigma(\gamma\gamma \rightarrow \pi^0 \pi^0)$ around the $f_0(980)$ resonance is quite sensitive to this constant. We impose that $\sigma(\gamma\gamma \rightarrow \pi^0 \pi^0) \leq 40$ nb at s_1 . This upper bound for the peak of the $f_0(980)$ in $\gamma\gamma \rightarrow \pi^0 \pi^0$ is equivalent to impose that the $\gamma\gamma$ width of the $f_0(980)$ lies in the range $205_{-83}^{+95}(\text{stat})_{-117}^{+147}(\text{sys})$ eV as determined in ref.[13]. We shall see that the effect of this rather large uncertainty allowed at 1 GeV, see fig.1, is very mild at lower energies. As the $f_0(980)$ resonance gives rise to a small *peak* in the precise data on $\gamma\gamma \rightarrow \pi^+ \pi^-$ [13], then $\phi_0(s)$ must increase with energy above s_K and the case with $z = +1$ is the one realized in nature. Note that for $z = -1$ in eq.(2), there is no a local maximum associated with this resonance in $|F_0(s)|$ but a minimum, because $|\omega_0(s)|$ has a dip around the $f_0(980)$ mass.

The source of uncertainty in the approximate relation $\phi_0(s) \simeq \delta_\pi(s)_0$ for $4m_K^2 \lesssim \sqrt{s} \lesssim 1.5$ GeV and its functional dependence for $s > s_H = 2.25$ GeV² is estimated similarly as in ref.[5, 8]. In fig.1 we show our final results for the $\gamma\gamma \rightarrow \pi^0 \pi^0$, where the band around each line corresponds to the estimated error. The error band for the dot-dashed line is not shown because it is similar to the ones of the other two curves. In this figure PY refers to using the $I = 0$ S-wave $\pi\pi$ of ref.[14], CGL that of ref.[15] and AO the one of ref.[2]. One observes that for $\sqrt{s} \lesssim 0.8$ GeV the uncertainty in the loose bound for the $f_0(980)$ greatly disappears. For such energies the main source of uncertainty originates from the uncertainties in the $\pi\pi$ phase parameterizations used.

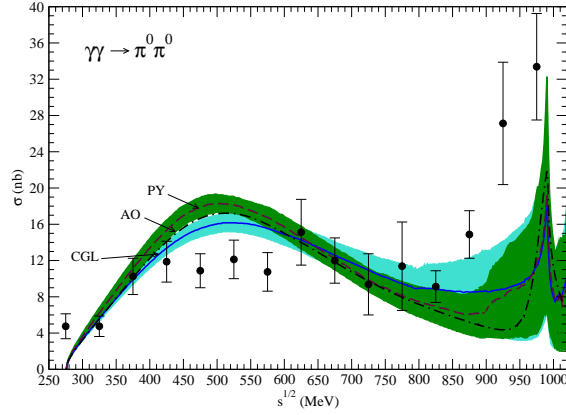


FIGURE 1. Final result for the $\gamma\gamma \rightarrow \pi^0\pi^0$ cross for $\sqrt{s} \leq 1.05$ GeV. The experimental data are from the Crystal Ball Collaboration [16].

The $\sigma \rightarrow \gamma\gamma$ width

For the evaluation of the coupling $\sigma \rightarrow \gamma\gamma$, $g_{\sigma\gamma\gamma}$, the amplitude $F_N(s)$ has to be evaluated on the second Riemann sheet, with $q_\pi \rightarrow -q_\pi$. We denote by $\tilde{F}_0(s)$ and $T_{II}(s)$ the $I = 0$ S-wave amplitudes evaluated on this sheet. Both have a pole corresponding to the σ resonance at s_σ , so that

$$T_{II}^{I=0} = -\frac{g_{\sigma\pi\pi}^2}{s_\sigma - s}, \quad \tilde{F}_0(s) = \sqrt{2} \frac{g_{\sigma\gamma\gamma} g_{\sigma\pi\pi}}{s_\sigma - s}, \quad (6)$$

where $g_{\sigma\pi\pi}$ is the coupling of the σ to $\pi\pi$. The relation between F_0 and \tilde{F}_0 can be easily established using unitarity above the $\pi\pi$ threshold [4, 5] so that $\tilde{F}_0(s) = F_0(s) (1 + 2i\rho(s)T_{II}^{I=0}(s))$. Taking this into account together with eq.(6) it follows that

$$\frac{g_{\sigma\gamma\gamma}^2}{g_{\sigma\pi\pi}^2} = -\frac{1}{2} \left(\frac{\sigma_\pi(s_\sigma)}{8\pi} \right)^2 F_0(s_\sigma)^2. \quad (7)$$

We denote by $s_\sigma = (M_\sigma - i\Gamma_\sigma/2)^2$. Ref.[1] provides $M_\sigma^{CCL} = 441_{-8}^{+16}$ MeV and $\Gamma_\sigma^{CCL} = 544_{-25}^{+18}$ MeV, while from ref.[2] one has $M_\sigma^{AO} = (456 \pm 6)$ MeV and $\Gamma_\sigma^{AO} = (482 \pm 20)$ MeV. In the following the superscripts *AO* and *CCL* refer to those results obtained by employing s_σ from ref.[2] or [1], respectively. From eq.(7) we obtain $|g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}| = 2.01 \pm 0.11$ for s_σ^{CCL} and 1.85 ± 0.09 for s_σ^{AO} . Given s_σ , this ratio of residues is the well defined prediction that follow from our $F_0(s)$. We employ the standard narrow resonance width formula in terms of $g_{\sigma\gamma\gamma}$ to calculate $\Gamma(\sigma \rightarrow \gamma\gamma) = \frac{|g_{\sigma\gamma\gamma}|^2}{16\pi M_\sigma}$. One needs to provide numbers for $|g_{\sigma\pi\pi}|$ in order to apply the previous equation and the determined $|g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|$. We first consider the value $|g_{\sigma\pi\pi}^{AO}| = (3.17 \pm 0.10)$ GeV from the approach of ref.[2]. The calculated width is $\Gamma^{AO}(\sigma \rightarrow \pi\pi) = (1.50 \pm 0.18)$ KeV. Not only the position of the pole in the partial wave amplitude, but also its residue can be calculated in the framework of the dispersive analysis described in ref.[1]. Expressed in terms of

the complex coefficient $g_{\sigma\pi\pi}$ defined in eq.(6), the preliminary result for the residue amounts to $|g_{\sigma\pi\pi}^{CCL}| = (3.31_{-0.08}^{+0.17})$ GeV, $\Gamma^{CCL}(\sigma \rightarrow \gamma\gamma) = (1.98_{-0.24}^{+0.30})$ KeV. Taking the average between these two values for $\Gamma(\sigma \rightarrow \gamma\gamma)$ we end with,

$$\Gamma(\sigma \rightarrow \gamma\gamma) = (1.68 \pm 0.15) \text{ KeV} . \quad (8)$$

ON THE RATIO OF COUPLINGS $f_0(980) \rightarrow \pi\pi/f_0(980) \rightarrow K\bar{K}$

In ref.[17] the $I = 0$ and 1 S-wave meson-meson amplitudes were studied using low-order Chiral Perturbation Theory (CHPT) to provide the interaction kernels which were then implemented in a Bethe-Salpeter equation. With only one free parameter the resonances σ , $f_0(980)$ and $a_0(980)$ were generated and the scattering data reproduced properly. This paper was the basis for all the later developments in Unitary CHPT. According to this approach, the $I = 0$ S-wave with the $\pi\pi$ and $K\bar{K}$ channels can be written in matrix notation as:

$$T = [I + V \cdot G]^{-1} \cdot V , \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix} , \quad G = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix} , \quad (9)$$

with $V_{11} = (s - m_\pi^2/2)/f_\pi^2$, $V_{12} = \sqrt{3}s/4f_\pi^2$, $V_{22} = 3s/4f_\pi^2$ and the labels 1 and 2 refer to the $I = 0$ $\pi\pi$ and $K\bar{K}$ states, in order. The function $g_i(s)$ is given by

$$g_i(s) = \frac{1}{16\pi^2} \left(\alpha_i + \log \frac{m_i^2}{\mu^2} - \sigma_i(s) \log \frac{\sigma_i(s) - 1}{\sigma_i(s) + 1} \right) , \quad (10)$$

where $\sigma_i(s) = \sqrt{1 - 4m_i^2/s}$. The constant α_i can be fixed by comparing the previous expression with the one calculated in terms of a three-momentum cut-off, the latter can be found in ref.[18]. One has then $\alpha_i = -\log(1 + \sqrt{1 + m_i^2/\Lambda_\chi^2})^2 + \log \mu^2/\Lambda_\chi$, with $\Lambda_\chi \simeq 1$ GeV, the scale of the CHPT suppression parameter. On the other hand, m_1 and m_2 are the pion and kaon masses, respectively. Taking the ratio of T_{12} and T_{22} from eq.(9) one has:

$$\frac{T_{12}}{T_{22}} = \frac{1/\sqrt{3}}{1 + g_1 3s/4f^2} . \quad (11)$$

Since $s \simeq 1$ GeV² we have neglected the factor $2m_1^2 \ll 3s$ in $3s - 2m_1^2$. When inverting the matrix $[I + V \cdot G]$ in eq.(9) the zeroes of its determinant determine the positions of the resonance poles. This determinant is given by $1 + V_{22}g_2 + V_{11}g_1 + (V_{11}V_{22} - V_{12}^2)g_1g_2$ from where at the pole position, s_R , one has $V_{22}(s_R) = -\frac{g_1}{g_2} [1 - V_{12}^2 g_1 g_2 / (1 + V_{11} g_1)]$. This expression can be substituted in eq.(11) because $3s/4f^2 = V_{22}$, with the result,

$$\frac{T_{12}}{T_{22}} = \frac{1/\sqrt{3}}{1 - \frac{g_1}{g_2} (1 - V_{12}^2 g_1 g_2 / (1 + V_{11} g_1))} . \quad (12)$$

Now eq.(9) for T is equivalent to $T = V + V \cdot G \cdot T$. As $V \propto f^{-2} \propto N_c^{-1}$ and $g_i = \mathcal{O}(N_c^0)$, with N_c the number of colours, it is necessary that the pole positions of the σ , $f_0(980)$

(and also for the $a_0(980)$) run as f^2 . In this way, $V_{ij}(s_R) \propto s_R/f^2 = \mathcal{O}(N_c^0)$ and there is no a mismatch in the way that T runs with N_c at s_R between the left and right sides of the previous expression. The same result was obtained in ref.[19] considering directly the dependence on N_c of the solutions of the equation $1 + V \cdot G = 0$. Thus, eq.(12) is $\mathcal{O}(N_c^0)$. It is worth stressing that this equation corresponds to the ratio of the $f_0(980)$ couplings to $\pi\pi$ and $K\bar{K}$, γ_π and γ_K , respectively, when evaluated at the $f_0(980)$ pole position. As a result this ratio does not run with N_c and the suppression of the $\pi\pi$ coupling compared to the $K\bar{K}$ one, around a factor 3 smaller as given by eq.(12), does not stem from the OZI rule. Let us recall that within QCD the OZI rule is a requirement of the large N_c limit. This suppression of the $\pi\pi$ coupling originates from the factor $1/\sqrt{3}$ in eq.(12), required by the expressions of V_{ij} from lowest order CHPT, and from the rescattering, accounted for by the denominator in eq.(12).

In summary, we have presented a brief account of the results of refs.[4, 5] on the calculation of $\gamma\gamma \rightarrow \pi\pi$ from dispersion relations and of the width $\Gamma(\sigma \rightarrow \gamma\gamma)$. In addition, we have shown that the ratio of the $f_0(980)$ couplings to $\pi\pi$ and $K\bar{K}$ does not run with N_c , so that the suppression of the former compared with the latter has nothing to do with the OZI rule.

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